Calculus I review- Chapters 1-5
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I. LIMITS Find the following limits. If the limit does not exist, say so, and give reason for credit.

(a) $\lim_{x\to\infty} \frac{3x^2+7x-1}{5x^2+\frac{79}{x}-391}$
(b) $\lim_{x\to\infty} \sqrt{x^2-1}-x$
(c) $\lim_{x\to 0} \frac{sin3x}{2x}$
(d) $\lim_{x\to -2} \frac{x-2}{x+2}$
(e) $\lim_{x\to 1} \frac{x^2+x-2}{x^2-3x+2}$
(f) Let $f(x) = \left\{ \begin{array}{cc} x+2 & 0 \le x < 3 \\ a-x^2 & 3 \le x \le 5 \end{array} \right\}$. Find the number a so that $\lim_{x\to 3} f(x)$ exists and find the limit. Once you do this, answer the question, "Is f(x) continuous at 3?"
(g) Tell if the functions listed below are continuous at the indicated point and which are not. Explain your answer.
 (i) $f(x) = x^2$ at $x = -1$.
 (ii) $f(x) = \left\{ \begin{array}{cc} \dfrac{x^2-9}{x+3} & x \neq -3 \\ 0 & x = -3 \end{array} \right\}$ at $x = -3$.
 (iii) $f(x) = \frac{x^2-9}{x+3}$ at $x = -3$.
 (iv) $f(x) = \frac{1}{x-2}$ at $x = 2$.

II. DERIVATIVES Differentiate each function.

(a) $f(x) = sin(^3\sqrt{x^2+tanx})$
(b) $g(x) = 3^x \cdot sin(cosx)$
(c) $h(x) = \int_2^{x^2} \frac{1}{sinu+\sqrt{u}}du, 0 < x < \frac{\pi}{2}$.
(d) $g(x) = \frac{sin^2 x}{sin(x^2)}$
(e) Find $\frac{d^5}{dx^5}$ for $f(x) = x^4 - 6x + 1$
(f) Graph the following functions showing all information we can derive from derivatives, etc. (i.e., c.p.'s, max, min, concavity, inflection points, asymptotes, special properties, intercepts..)
(1) $f(x) = x^3 - 6x^2 + 9x$
(2) $g(x) = \frac{4}{1-x^2}$.
(g) Calculate the derivative of $f(x) = \frac{1}{x}$ using the definition of the derivative. Check your answer using the power rule. BE SURE TO STATE THE DEFINITION.
(h) Find $\frac{dy}{dx}$ for $xy = siny$.
(i) Find the equation of the tangent line to the above curve (in part (h)) at the point $(\frac{2}{\pi}, \frac{\pi}{2})$.

III. INTEGRATION Evaluate the following DEFINITE and INDEFINITE integrals.

(a) $\int x^2(x^3+2)^{\frac{1}{5}}\ dx$
(b) $\int \frac{1}{\sqrt{x}} cos(\sqrt{x})\ dx$
(c) $\int_0^1 \frac{x^4+4}{x^5+20x+1}\ dx$
(d) $\int \frac{e^{2t}}{\sqrt[3]{e^{2t}+1}}\ dt$
(e) Find the area of the region enclosed by the curves $x=-1,\ y=x^3-6x^2+9x,$ $y=x,\ x=5$. (Hint: does this function look familiar?)
(f) Find the volumes for the following regions generated by rotating the given functions about the given lines. Try to use the disk method and cylindrical shell method each once.
  (1) $y=3x-3,\ x=1, x=3, y=0$ about the y-axis.
  (2) $y=x^3$ and $y=2x^2$, about the x-axis.

IV. APPLICATIONS

(a) Coca-cola wants to design a new 1 liter can in the usual shape of a right circular cylinder (with top and bottom). What dimensions will use the least amount of material?
Volume $=\pi r^2 h$, Surface area $=2\pi rh+2\pi r^2$
(b) An electron moves about a nucleus (centered at (0,0)), in a perfect circular motion with radius 3. When the electron is at the point $(-\sqrt{3},\sqrt{6})$, its x-direction coordinate is increasing at a rate of 20 units per second. How fast is the y-direction coordinate changing at that instant?
(c) $v(t)=t^2-6t+1$. Find an expression for both the position and acceleration functions of the particle if the initial position, s(0), is 4.