

APPENDIX A - *HOSTSIM* TECHNICAL DESCRIPTION

This document contains the equations for each compartment in *HostSim*, how they are coupled, parameter ranges, and initial conditions. *HostSim* was originally published by Louis Joslyn, Jennifer Linderman, and Denise Kirschner in the article “A virtual host model of Mycobacterium tuberculosis infection identifies early immune events as predictive of infection outcomes” in 2022 in the *Journal of Theoretical Biology*. This technical description is, in part, an updated reproduction of the supplemental material presented in 2024 by C.T. Michael et al. in the article “A framework for multi-scale intervention modeling: virtual cohorts, virtual clinical trials, and model-to-model comparisons” in *Frontiers in Systems Biology*.

We have separated this document into sections for readability. In Section 1, we detail all of the equations in each granuloma compartment. In Section 2, we detail the equations in the lymph node and blood compartments. In Section 3, we present the method by which all of our compartments are coupled. In Section 4, we describe how we model dissemination - that is, how we represent one granuloma causing another to form. In Section 5, we present tables for the parameter ranges and initial condition values for our granuloma (Section 5.1), lymph node and blood (Section 5.2), and pharmacokinetic (Section 5.3) model components. In each of these sections, we included both a symbolic variable for each variable (e.g., M_R or ξ_2) and a corresponding non-symbolic variable (e.g., MR or xi2) for convenience of navigating this document via text search.

1. GRANULOMA EQUATIONS

Each granuloma has its own distinctly-parameterized instance of the equations presented in the subsections below. Each host has a “host-scale granuloma parameter base” selected by the LHS sampling scheme using the ranges presented in Section 5.1. To capture intra-host variability, individual granulomas within a host are given parameter values normally distributed around this base host value. Terms indicated by *Pulled* $[\cdot]_g$ involve quantities pulled from the lymph node and blood equations, and is described in Section 3. Below is an index of all state variables in granulomas.

State Variable	Non-Symbolic	Units	Description
M_R	MR	count	Resting macrophages
M_I	MI	count	Infected macrophages
M_A	MA	count	Activated macrophages
B_I	BI	count	Intracellular Mtb
B_E	BE	count	Extracellular Mtb
B_N	BN	count	Non-replicating Mtb
Ca	CA	mass of $M\phi$	Necrotic tissue mass
Ag	AG	mass Ag/Mtb	Antigen mass within granulomas
T_0^4	T40	count	Mtb-specific primed $CD4^+$ T-cells
T_1^4	Th1	count	Mtb-specific Th1 $CD4^+$ T-cells
T_2^4	Th2	count	Mtb-specific Th2 $CD4^+$ T-cells
T_{Non}^4	CD4Non	count	Nonspecific $CD4^+$ T-cells
T_{EM}^4	EMCD4	count	Mtb-specific $CD4^+$ effector memory T-cells
T_0^8	T80	count	Mtb-specific primed $CD8^+$ T-cells
T_C	TC	count	Mtb-specific cytotoxic $CD8^+$ T-cells
T^8	T8	count	Mtb-specific effector $CD8^+$ T-cells
T_{EM}^8	EMCD8	count	Mtb-specific $CD8^+$ effector memory T-cells
T_{Non}^8	CD8Non	count	Nonspecific $CD8^+$ T-cells
F_α	TNF	pg/mL	TNF- α concentration
I_γ	IG	pg/mL	IFN- γ concentration
I_{12}	I12	pg/mL	Interleukin 12 concentration
I_{10}	I10	pg/mL	Interleukin 10 concentration
I_4	I4	pg/mL	Interleukin 4 concentration

1.1. Macrophage equations.

M_R - MR - Resting macrophage count

$$\begin{aligned}
\frac{d}{dt}(M_R) = & \underbrace{\alpha_{4a}(M_A + w_2 M_I)}_{\text{Macrophage-driven recruitment}} + \underbrace{S r_{4b} \left(\frac{F_\alpha}{F_\alpha + f_8 I_{10} + s_{4b}} \right)}_{\text{TNF driven recruitment}} + \underbrace{\psi_C \psi_S \left(\sum_{j=1}^n R_{IC/EC}^{FIC} \text{ for drug j} \right)}_{\text{Drug-induced reversion from } M_I}^{1/FIC} k_I(C_{eff}) M_I \\
& - \underbrace{k_2 M_R \left(\frac{B_E}{B_E + c_9} \right)}_{\text{Infection of } M_R} - \underbrace{k_3 M_R \left(\frac{B_E + w B_I + \beta F_\alpha}{B_E + w B_I + \beta F_\alpha + c_8} \right) \left(\frac{I_\gamma}{I_\gamma + f_1 I_4 + f_7 I_{10} + s_1} \right)}_{\text{Activation of macrophages}} - \underbrace{\mu_{M_R} M_R}_{\text{Natural death}}
\end{aligned}$$

M_I - M_I - Infected macrophages

$$\begin{aligned}
\frac{d}{dt}(M_I) = & \underbrace{k_2 M_R \left(\frac{B_E}{B_E + c_9} \right)}_{\text{Infection of } M_R} - \underbrace{k_{17} M_I \left(\frac{B_I^2}{B_I^2 + (N M_I)^2} \right)}_{\text{Bursting of } M_I} - \underbrace{k_{14a} M_I \left(\frac{\left(\frac{T_C + w_3 T_1^4}{M_I} \right)}{\left(\frac{T_C + w_3 T_1^4}{M_I} \right) + c_4} \right)}_{\text{T-cell driven apoptosis of } M_I} \\
& - \underbrace{k_{14b} M_I \left(\frac{F_\alpha}{F_\alpha + f_9 I_{10} + s_{4b}} \right)}_{\text{TNF driven apoptosis of } M_I} - \underbrace{k_{52} M_I \left(\frac{\left(\frac{T_C \left(\frac{T_1^4}{T_1^4 + C_{T_1^4}} \right) + w_1 T_1^4}{M_I} \right)}{\left(\frac{T_C \left(\frac{T_1^4}{T_1^4 + C_{T_1^4}} \right) + w_1 T_1^4}{M_I} \right) + c_{52}} \right)}_{\text{Cytotoxic T-cell driven } M_I \text{ apoptosis}} \\
& - \underbrace{\psi_C \left(\sum_{j=1}^n R_{IC/EC}^{FIC} \text{ for drug j} \right)}_{\text{Drug-induced clearance}}^{1/FIC} k_I(C_{eff}) M_I - \underbrace{\mu_{M_I} M_I}_{\text{Natural death}}
\end{aligned}$$

M_A - M_A - Activated macrophages

$$\begin{aligned}
\frac{d}{dt}(M_A) = & \underbrace{k_3 M_R \left(\frac{B_E + w B_I + \beta F_\alpha}{B_E + w B_I + \beta F_\alpha + c_8} \right) \left(\frac{I_\gamma}{I_\gamma + f_1 I_4 + f_7 I_{10} + s_1} \right)}_{\text{Activation of resting macrophages}} - \underbrace{k_4 M_A \left(\frac{I_{10}}{I_{10} + s_8} \right)}_{\text{Deactivation by } I_{10}} - \underbrace{\mu_{M_A} M_A}_{\text{Natural death}}
\end{aligned}$$

1.2. Mtb and caseum equations.

B_I - B_I - Intracellular bacterium count

$$\begin{aligned}
\frac{d}{dt}(B_I) = & \underbrace{\alpha_{19} \frac{B_I}{M_I} M_I \left(1 - \frac{(B_I/M_I)}{N}\right)}_{\text{Replication}} + \underbrace{k_2 \frac{N}{2} M_R \left(\frac{B_E}{B_E + c_9}\right)}_{\text{Internalization of } B_E} - \underbrace{k_{17} N M_I \left(\frac{B_I^2}{B_I^2 - (N M_I)^2}\right)}_{\text{Bursting of } M_I} \\
& - \underbrace{k_{14a} \frac{B_I}{M_I} M_I \left(\frac{\left(\frac{T_C + w_3 T_1^4}{M_I}\right)}{\left(\frac{T_C + w_3 T_1^4}{M_I}\right) + c_4}\right)}_{\text{T-cell driven } M_I \text{ apoptosis}} - \underbrace{k_{14b} \frac{B_I}{M_I} M_I \left(\frac{F_\alpha}{F_\alpha + f_9 I_{10} + s_{4b}}\right)}_{\text{TNF driven apoptosis}} \\
& - \underbrace{k_{52} \frac{B_I}{M_I} M_I \left(\frac{\left(\frac{T_C \left(\frac{T_1^4}{T_1^4 + C_{T_1^4}}\right) + w_1 T_1^4}{M_I}\right)}{\left(\frac{T_C \left(\frac{T_1^4}{T_1^4 + C_{T_1^4}}\right) + w_1 T_1^4}{M_I}\right) + c_{52}}\right)}_{\text{Cytotoxic T-cell driven } M_I \text{ apoptosis}} - \underbrace{\mu_{B_I} B_I}_{\text{Natural death}} - \underbrace{\mu_{M_I} \frac{B_I}{M_I} M_I}_{\text{Natural death of } M_I} - \underbrace{k_I (C_{eff}) B_I}_{\text{Drug killing of } B_I \text{ based on effective killing rate}}
\end{aligned}$$

B_E - BE - Extacellular bacterium count

$$\begin{aligned}
\frac{d}{dt}(B_E) = & \underbrace{\alpha_{20} B_E \left(1 - \frac{B_E}{10^6}\right)}_{\text{Replication}} + \underbrace{(1 - C_N)}_{\text{Fraction outside of caseum}} \left[\underbrace{k_{17} N M_I \left(\frac{B_I^2}{B_I^2 + (N M_I)^2}\right)}_{\text{Macrophage bursting}} + \underbrace{k_{14a} N_{fracc} \frac{B_I}{M_I} M_I \left(\frac{\left(\frac{T_C + w_3 T_1^4}{M_I}\right)}{\left(\frac{T_C + w_3 T_1^4}{M_I}\right) + c_4}\right)}_{\text{T-cell driven apoptosis}} \right. \\
& \left. + \underbrace{k_{14b} N_{fraca} \frac{B_I}{M_I} M_I \left(\frac{F_\alpha}{F_\alpha + f_9 I_{10} + s_{4b}}\right)}_{\text{TNF driven apoptosis}} + \underbrace{\mu_{M_I} N_{fracd} \frac{B_I}{M_I} M_I}_{\text{Natural death of } M_I} \right] + \underbrace{k_{Rev} N_{Ca} M_A C_a \cdot B_N}_{\text{Revealing of } B_N} \\
& - \underbrace{k_2 \frac{N}{2} M_R \left(\frac{B_E}{B_E + c_9}\right)}_{\text{Internalization of } B_E} - \underbrace{k_{15} M_A B_E}_{M_A \text{ killing of } B_E} - \underbrace{k_{18} M_R B_E}_{M_R \text{ killing of } B_E} - \underbrace{\mu_{B_E} B_E}_{\text{Natural death}} - \underbrace{k_E (C_{eff}) B_E}_{\text{Drug killing of } B_E \text{ based on effective killing rate}}
\end{aligned}$$

B_N - BN - Nonreplicating bacterium count

$$\begin{aligned}
\frac{d}{dt}(B_N) = & \underbrace{C_N}_{\text{Fraction inside of caseum}} \left\{ \underbrace{k_{17} M_I \left(\frac{B_I^2}{B_I^2 + (N M_I)^2}\right)}_{\text{Macrophage bursting}} + \underbrace{k_{14a} N_{fracc} \frac{B_I}{M_I} M_I \left(\frac{\left(\frac{T_C + w_3 T_1^4}{M_I}\right)}{\left(\frac{T_C + w_3 T_1^4}{M_I}\right) + c_4}\right)}_{\text{T-cell driven apoptosis}} \right. \\
& \left. + \underbrace{k_{14b} N_{fraca} \frac{B_I}{M_I} M_I \left(\frac{F_\alpha}{F_\alpha + f_9 I_{10} + s_{4b}}\right)}_{\text{TNF driven apoptosis}} + \underbrace{\mu_{M_I} N_{fracd} \frac{B_I}{M_I} M_I}_{\text{Natural death of } M_I} \right\} \\
& - \underbrace{\mu_{B_N} B_N}_{\text{Natural death}} - \underbrace{B_N k_{Rev} N_{Ca} C_a \cdot M_A}_{\text{Revealing of } B_N} - \underbrace{k_N (C_{eff}) B_N}_{\text{Drug killing of } B_N \text{ based on effective killing rate}}
\end{aligned}$$

C_a - CA - Caseum

$$\begin{aligned}
N_f \frac{d}{dt} (Ca) = & \underbrace{k_{17} M_I \left(\frac{B_I^2}{B_I^2 + (N M_I)^2} \right)}_{\text{Bursting of } M_I} + \underbrace{k_{14a} M_I \left(\frac{\left(\frac{T_C + w_3 T_1}{M_I} \right)}{\left(\frac{T_C + w_3 T_1}{M_I} \right) + c_4} \right)}_{\text{T-cell mediated apoptosis of } M_I} + \underbrace{k_{14b} M_I \left(\frac{F_\alpha}{F_\alpha + f_9 I_{10} + s_{4b}} \right)}_{\text{TNF-mediated apoptosis of } M_I} \\
& + \underbrace{k_{52} \frac{B_I}{M_I} M_I \left(\frac{\left(\frac{T_C \left(\frac{T_1^4}{T_1^4 + C_{T_1^4}} \right) + w_1 T_1^4}{M_I} \right)}{\left(\frac{T_C \left(\frac{T_1^4}{T_1^4 + C_{T_1^4}} \right) + w_1 T_1^4}{M_I} \right) + c_{52}} \right)}_{\text{Cytotoxic T-cell mediated apoptosis of } M_I} + \underbrace{\psi_C (1 - \psi_S) \left(\sum_{j=1}^n R_{IC/EC}^{FIC} \text{ for drug } j \right)^{1/FIC} k_I (C_{eff}) M_I}_{\text{M}_I \text{ killed during drug clearance}} \\
& + \underbrace{\mu_{M_I} M_I + \mu_{M_A} M_A + \mu_{M_R} M_R \left(\frac{F_\alpha}{F_\alpha + f_9 I_{10} + s_{4b}} \right)}_{\text{Natural death of all macrophage populations}} - \underbrace{\frac{N_{Ca} C_a M_A}{N_{Ca} C_a M_A}}_{\text{Neutrophil-driven caseum clearance}}
\end{aligned}$$

Ag - AG - Mtb antigen

$$\begin{aligned}
\frac{d}{dt} (Ag) = & \underbrace{k_{15} M_A B_E}_{M_A \text{ killing of } B_E} + \underbrace{k_{18} M_R B_E}_{M_R \text{ killing of } B_E} + \underbrace{\mu_{B_E} B_E}_{\text{Natural death of } B_E} \\
& + \underbrace{k_I (C_{eff}) B_I}_{\text{Drug killing of } B_I \text{ based on effective killing rate}} + \underbrace{k_E (C_{eff}) B_E}_{\text{Drug killing of } B_E \text{ based on effective killing rate}} + \underbrace{k_N (C_{eff}) B_N}_{\text{Drug killing of } B_N \text{ based on effective killing rate}} \\
& + \underbrace{k_{14a} \frac{B_I}{M_I} M_I \left(\frac{\left(\frac{T_C + w_3 T_1}{M_I} \right)}{\left(\frac{T_C + w_3 T_1}{M_I} \right) + c_4} \right)}_{\text{T-cell driven } M_I \text{ apoptosis}} + \underbrace{k_{14b} \frac{B_I}{M_I} M_I \left(\frac{F_\alpha}{F_\alpha + f_9 I_{10} + s_{4b}} \right)}_{\text{TNF driven apoptosis}} \\
& + \underbrace{k_{52} \frac{B_I}{M_I} M_I \left(\frac{\left(\frac{T_C \left(\frac{T_1^4}{T_1^4 + C_{T_1^4}} \right) + w_1 T_1^4}{M_I} \right)}{\left(\frac{T_C \left(\frac{T_1^4}{T_1^4 + C_{T_1^4}} \right) + w_1 T_1^4}{M_I} \right) + c_{52}} \right)}_{B_I \text{ death by cytotoxic T-cell driven } M_I \text{ apoptosis}} + \underbrace{\mu_{B_I} B_I}_{\text{Natural Death of } B_I} \\
& + \underbrace{\mu_{M_I} \frac{B_I}{M_I} M_I}_{B_I \text{ death by natural death of } M_I} - \underbrace{\mu_{Ag} Ag}_{\text{Antigen clearance}}
\end{aligned}$$

1.3. CD4⁺ T-cells.

T_0^4 - T40 - Primed CD4⁺ T-cell count

$$\begin{aligned} \frac{d}{dt} (T_0^4) = & \underbrace{Pulled [T_0^4]_g + \alpha_2 T_0^4 \left(\frac{M_A}{M_A + c_{15}} \right)}_{\text{Proliferation}} - \underbrace{k_6 I_{12} T_0^4 \left(\frac{I_\gamma}{I_\gamma + f_1 I_4 + f_7 I_{10} + s_1} \right)}_{\text{Differentiation to } T_1^4} \\ & - \underbrace{k_7 T_0^4 \left(\frac{I_4}{I_4 + f_2 I_\gamma + s_2} \right)}_{\text{Differentiation to } T_2^4} - \underbrace{\mu_{T_0} T_0}_{\text{Natural death}} \end{aligned}$$

T_1^4 - Th1 - Effector CD4⁺ T-cell count

$$\begin{aligned} \frac{d}{dt} (T_1^4) = & \underbrace{k_6 I_{12} T_0^4 \left(\frac{I_\gamma}{I_\gamma + f_1 I_4 + f_7 I_{10} + s_1} \right)}_{\text{Differentiation from } T_0^4} + \underbrace{k_{31} T_{EM}^4 M_I \left(\frac{M_I}{M_I + 3} \right)}_{\text{Differentiation from } T_{EM}^4} \\ & - \underbrace{\mu_{T_\gamma} \left(\frac{I_\gamma}{I_\gamma + c} \right) T_1^4 M_A}_{\text{IFN}\gamma \text{ apoptosis of } T_1^4} - \underbrace{\mu_{T_1} T_1^4}_{\text{Natural death}} \end{aligned}$$

T_2^4 - Th2 - Effector Th2 CD4⁺ T-cell count

$$\begin{aligned} \frac{d}{dt} (T_2^4) = & \underbrace{k_7 T_0^4 \left(\frac{I_4}{I_4 + f_2 I_\gamma + s_2} \right)}_{\text{Differentiation from } T_0^4} + \underbrace{k_{32} T_{EM}^4 M_A}_{\text{Differentiation from } T_{EM}^4} - \underbrace{\mu_{T_2} T_2^4}_{\text{Natural death}} \end{aligned}$$

T_{EM}^4 - EMCD4 - Effector memory CD4⁺ T-cell count

$$\begin{aligned} \frac{d}{dt} (T_{EM}^4) = & \underbrace{Pulled [T_{EM}^4]_g - k_{31} T_{EM}^4 M_I \left(\frac{M_I}{M_I + 3} \right)}_{\text{Differentiation to } T_1^4} - \underbrace{k_{32} T_{EM}^4 M_A}_{\text{Differentiation to } T_2^4} - \underbrace{\mu_{T_{EM}} T_{EM}^4}_{\text{Natural death}} \end{aligned}$$

T_{Non}^4 - CD4Non - Non-cognate CD4⁺ T-cell count

$$\frac{d}{dt} (T_{Non}^4) = \underbrace{Pulled [T_{Non}^4]_g - \mu_{T_{Non}} T_{Non}^4}_{\text{Natural death}}$$

1.4. CD8⁺ T-cells.

T_0^8 - T80 - Primed CD8⁺ T-cell count:

$$\begin{aligned} \frac{d}{dt} (T_0^8) = & \underbrace{Pulled [T_0^8]_g + \alpha_2 T_0^8 \left(\frac{M_A}{M_A + c_{15}} \right)}_{\text{Proliferation}} \\ & - \underbrace{k_6 I_{12} T_0^8 \left(\frac{I_\gamma}{I_\gamma + f_1 I_4 + f_7 I_{10} + s_1} \right)}_{\text{Differentiation to } T^8 \text{ and } T_C} - \underbrace{\mu_{T_0} T_0^8}_{\text{Natural death}} \end{aligned}$$

T^8 - T8 - Effector CD8⁺ T-cell count

$$\begin{aligned} \frac{d}{dt} (T^8) = & \underbrace{(m) k_6 I_{12} T_0^8 \left(\frac{I_\gamma}{I_\gamma + f_1 I_4 + f_7 I_{10} + s_1} \right)}_{\text{Differentiation from } T_0^8} + \underbrace{k_{34} T_{EM}^8 M_I \left(\frac{M_I}{M_I + 3} \right)}_{\text{Differentiation from } T_{EM}^8} \\ & - \underbrace{\mu_{T_C \gamma} \left(\frac{I_\gamma}{I_\gamma + c_c} \right) T^8 M_A}_{\text{IFN}\gamma \text{ apoptosis}} - \underbrace{\mu_{T_C} T^8}_{\text{Natural death}} \end{aligned}$$

T_C - TC - Cytotoxic CD8⁺ T-cell count

$$\frac{d}{dt}(T_C) = \underbrace{(1-m)k_6 I_{12} T_0^8 \left(\frac{I_\gamma}{I_\gamma + f_1 I_4 + f_7 I_{10} + s_1} \right)}_{\text{Differentiation from } T_0^8} + \underbrace{k_{33} T_{EM}^8 M_I \left(\frac{M_I}{M_I + 3} \right)}_{\text{Differentiation from } T_{EM}^8} - \underbrace{\mu_{T_C \gamma} \left(\frac{I_\gamma}{I_\gamma + c_c} \right) T_C M_A}_{\text{IFN}\gamma \text{ apoptosis}} + \underbrace{\mu_{T_C} T_C}_{\text{Natural death}}$$

T_{EM}^8 - EMCD8 - Effector Memory CD8⁺ T-cell count

$$\frac{d}{dt}(T_{EM}^8) = \text{Pulled}[T_{EM}^8]_g - \underbrace{k_{33} T_{EM}^8 M_I \left(\frac{M_I}{M_I + 3} \right)}_{\text{Differentiation to } T_C} - \underbrace{k_{34} T_{EM}^8 M_I \left(\frac{M_I}{M_I + 3} \right)}_{\text{Differentiation to } T^8} - \underbrace{\mu_{T_{EM}^8} T_{EM}^8}_{\text{Natural death}}$$

T_{Non}^8 - CD8Non - Non-cognate CD8⁺ T-cell count

$$\frac{d}{dt}(T_{Non}^8) = \text{Pulled}[T_{Non}^8]_g - \underbrace{\mu_{T_{Non}^8} T_{Non}^8}_{\text{Natural death}}$$

1.5. Cytokines.

F_α - TNF - concentration of Tumor Necrosis Factor α

$$\begin{aligned} \frac{d}{dt}(F_\alpha) = & \underbrace{\alpha_{30} M_I}_{\text{Production by } M_I} + \underbrace{\alpha_{31} M_A \left(\frac{I_\gamma + \beta_2 (B_E + w B_I)}{I_\gamma + \beta_2 (B_E + w B_I) + f_1 I_4 + f_7 I_{10} + s_{10}} \right)}_{\text{IFN-IL mediated production of TNF}\alpha \text{ by } M_A} \\ & + \underbrace{\alpha_{32} T_1^4}_{\text{Production by } T_1^4} + \underbrace{\alpha_{33} \left(\frac{T_C + T^8}{2m} \right)}_{\text{Production by CD8}^+ \text{ T-cells}} - \underbrace{\mu_{F_\alpha} F_\alpha}_{\text{Clearance}} \end{aligned}$$

I_γ - IG - concentration of Interferon- γ

$$\begin{aligned} \frac{d}{dt}(I_\gamma) = & \underbrace{s_g \left(\frac{B_E + w B_I}{B_E + w B_I + c_{10}} \right) \left(\frac{I_{12}}{I_{12} + s_7} \right)}_{\text{Other}} + \underbrace{\alpha_{5a} T_1^4 \left(\frac{M_A}{M_A + c_{5a}} \right)}_{\text{Production by } T_1^4} + \underbrace{\alpha_{5b} T^8 \left(\frac{M_A}{M_A + c_{5b}} \right)}_{\text{Production by } T^8} + \underbrace{\alpha_{5c} M_I}_{\text{Production by } M_I} \\ & + \underbrace{\alpha_7 T_0^4 \left(\frac{I_{12}}{I_{12} + f_4 I_{10} + s_4} \right) + \alpha_7 T_0^8 \left(\frac{I_{12}}{I_{12} + f_4 I_{10} + s_4} \right)}_{\text{Interleukin-mediated production by primed T-cells}} - \underbrace{\mu_{I_\gamma} I_\gamma}_{\text{Clearance}} \end{aligned}$$

I_{12} - I12 - concentration of Interleukin-12

$$\frac{d}{dt}(I_{12}) = \underbrace{s_{12} \left(\frac{B_E + w B_I}{B_E + w B_I + c_{230}} \right)}_{\text{Other}} + \underbrace{\alpha_{23} M_R \left(\frac{B_E + w B_I}{B_E + w B_I + c_{23}} \right)}_{\text{Production by } M_R} + \underbrace{\alpha_8 M_A \left(\frac{s}{s + I_{10}} \right)}_{\text{Production by } M_A} - \underbrace{\mu_{I_{12}} I_{12}}_{\text{Clearance}}$$

I_{10} - I10 - concentration of Interleukin-10

$$\frac{d}{dt}(I_{10}) = \underbrace{\delta_7 (M_I + M_A) \left(\frac{s_6}{I_{10} + f_6 I_\gamma + s_6} \right)}_{\text{IFN-IL mediated production by macrophages}} + \underbrace{\alpha_{16} T_1^4 + \alpha_{17} T_2^4}_{\text{Production by CD4}^+ \text{ T-cells}} + \underbrace{\alpha_{18} \left(\frac{T_C + T^8}{2m} \right)}_{\text{Production by CD8}^+ \text{ T-cells}} - \underbrace{\mu_{I_{10}} I_{10}}_{\text{Clearance}}$$

I_4 - I4 - concentration of Interleukin-4

$$\frac{d}{dt}(I_4) = \underbrace{\alpha_{11} T_0^4}_{\text{Production by } T_0^4} + \underbrace{\alpha_{12} T_2^4}_{\text{Production by } T_2^4} - \underbrace{\mu_{I_4} I_4}_{\text{Clearance}}$$

2. LYMPH NODE AND BLOOD EQUATIONS

The lymph node compartment will clonally expand and differentiate effector T cells to be sent to blood. Cells in blood can be recruited to granulomas based on their cytokine levels, which is represented by terms indicated by $Pulled[\cdot]_g$ (Section 3). We select each parameter in the following equations using the LHS sampling scheme using the ranges presented in Section 5.2. Below is an index of all state variables in the compartment representing lymph nodes and blood.

Variable	Non-symbolic	Unit	Variable description
Lymph Node			
APC	APC	count	Antigen-presenting cells in LN
N_4^{LN}	LnN4	count	Mtb-specific naive $CD4^+$ T-cell count in LN
P_4^{LN}	LnP4	count	Mtb-specific precursor $CD4^+$ T-cell count in LN
E_4^{LN}	LnE4	count	Mtb-specific effector $CD4^+$ T-cell count in LN
CM_4^{LN}	LnCM4	count	Mtb-specific central memory $CD4^+$ T-cell count in LN
EM_4^{LN}	LnEM4	count	Mtb-specific effector memory $CD4^+$ T-cell count in LN
$N_{Non,4}^{LN}$	LnN4Non	count	Nonspecific naive $CD4^+$ T-cell count in LN
$CM_{Non,4}^{LN}$	LnCM4Non	count	Nonspecific central memory $CD4^+$ T-cell count in LN
N_8^{LN}	LnN8	count	Mtb-specific naive $CD8^+$ T-cell count in LN
P_8^{LN}	LnP8	count	Mtb-specific precursor $CD8^+$ T-cell count in LN
E_8^{LN}	LnE8	count	Mtb-specific effector $CD8^+$ T-cell count in LN
CM_8^{LN}	LnCM8	count	Mtb-specific central memory $CD8^+$ T-cell count in LN
EM_8^{LN}	LnEM8	count	Mtb-specific effector memory $CD8^+$ T-cell count in LN
$N_{Non,8}^{LN}$	LnN8Non	count	Nonspecific naive $CD8^+$ T-cell count in LN
$CM_{Non,8}^{LN}$	LnCM8Non	count	Nonspecific central memory $CD8^+$ T-cell count in LN
Blood			
N_4^B	BIN4	pg/mL	Mtb-specific naive $CD4^+$ T-cell count in blood
E_4^B	BIE4	pg/mL	Mtb-specific effector $CD4^+$ T-cell count in blood
CM_4^B	BICM4	pg/mL	Mtb-specific central memory $CD4^+$ T-cell count in blood
EM_4^B	BIEM4	pg/mL	Mtb-specific effector memory $CD4^+$ T-cell count in blood
$N_{Non,4}^B$	BIN4Non	pg/mL	Nonspecific naive $CD4^+$ T-cell count in blood
$E_{Non,4}^B$	BIE4Non	pg/mL	Nonspecific effector $CD4^+$ T-cell count in blood
$CM_{Non,4}^B$	BICM4Non	pg/mL	Nonspecific central memory $CD4^+$ T-cell count in blood
$EM_{Non,4}^B$	BIEM4Non	pg/mL	Nonspecific effector memory $CD4^+$ T-cell count in blood
N_8^B	BIN8	pg/mL	Mtb-specific naive $CD8^+$ T-cell count in blood
E_8^B	BIE8	pg/mL	Mtb-specific effector $CD8^+$ T-cell count in blood
CM_8^B	BICM8	pg/mL	Mtb-specific central memory $CD8^+$ T-cell count in blood
EM_8^B	BIEM8	pg/mL	Mtb-specific effector memory $CD8^+$ T-cell count in blood
$N_{Non,8}^B$	BIN8Non	pg/mL	Nonspecific naive $CD8^+$ T-cell count in blood
$E_{Non,8}^B$	BIE8Non	pg/mL	Nonspecific effector $CD8^+$ T-cell count in blood

Variable	Non-symbolic	Unit	Variable description
$CM_{Non,8}^B$	BICM8Non	pg/mL	Nonspecific central memory $CD8^+$ T-cell count in blood
$EM_{Non,8}^B$	BIEM8Non	pg/mL	Nonspecific effector memory $CD8^+$ T-cell count in blood

2.1. Antigen Presenting Cells.

APC - APC - Antigen presenting cells. Received APCs are sent from granulomas, which is detailed in Section 3.

$$\frac{d}{dt}(APC) = -\mu_5 APC + \sum_{i \in \{\text{Granulomas}\}} \text{Received from Granuloma } i$$

2.2. CD4⁺ T-cells in lymph nodes.

N_4^{LN} - LnN4 - CD4⁺ Mtb-specific naive T-cell count

$$\frac{d}{dt}(N_4^{LN}) = \alpha \left[\underbrace{k_1 N_4^B \left(\frac{APC}{APC + hs_1} \right)}_{\text{Cytokine-driven recruitment}} + \underbrace{\xi_1 N_4^B}_{\text{LN Influx}} \right] - \underbrace{\xi_2 N_4^{LN}}_{\text{LN Efflux}} - \underbrace{k_2 N_4^{LN} APC}_{\text{Differentiation into } P_4^{LN}}$$

P_4^{LN} - LnP4 - CD4⁺ Mtb-specific precursor T-cell count

$$\begin{aligned} \frac{d}{dt}(P_4^{LN}) = & \underbrace{k_2 N_4^{LN} APC}_{\text{Differentiation from } N_4^{LN}} + \underbrace{k_3 CM_4^{LN} APC}_{\text{Differentiation from } CM_4^{LN}} + \underbrace{k_4 P_4^{LN} \left(1 - \frac{P_4^{LN}}{\rho_1} \right) \left(\frac{APC}{APC + hs_4} \right)}_{\text{Proliferation}} \\ & - \underbrace{k_5 P_4^{LN} \left(\frac{APC}{APC + hs_5} \right)}_{\text{Differentiation into } E_4^{LN}} - \underbrace{k_6 P_4^{LN} \left(1 - \frac{APC}{APC + hs_5} \right)}_{\text{Differentiation into } CM_4^{LN}} - \underbrace{\mu_6 P_4^{LN}}_{\text{Natural death}} \end{aligned}$$

E_4^{LN} - LnE4 - CD4⁺ Mtb-specific effector T-cell count

$$\frac{d}{dt}(E_4^{LN}) = \underbrace{k_5 P_4^{LN} \left(\frac{APC}{APC + hs_5} \right)}_{\text{Differentiation from } P_4^{LN}} - \underbrace{\xi_3 E_4^{LN}}_{\text{LN Efflux}} - \underbrace{k_7 E_4^{LN}}_{\text{Differentiation into } EM_4^{LN}}$$

CM_4^{LN} - LnCM4 - CD4⁺ Mtb-specific central memory T-cell count

$$\begin{aligned} \frac{d}{dt}(CM_4^{LN}) = & \alpha \left[\underbrace{k_8 CM_4^B \left(\frac{APC}{APC + hs_8} \right)}_{\text{Cytokine-mediated recruitment}} + \underbrace{\xi_4 CM_4^B}_{\text{LN Influx}} \right] + \underbrace{k_6 P_4^{LN} \left(1 - \frac{APC}{APC + hs_5} \right)}_{\text{Differentiation from } P_4^{LN}} \\ & - \underbrace{k_3 CM_4^{LN} APC}_{\text{Differentiation to } P_4^{LN}} - \underbrace{\xi_5 CM_4^{LN}}_{\text{LN Efflux}} \end{aligned}$$

EM_4^{LN} - LnEM4 - CD4⁺ Mtb-specific effector memory T-cell count

$$\frac{d}{dt}(EM_4^{LN}) = \underbrace{k_7 E_4^{LN}}_{\text{Differentiation from } E_4^{LN}} - \underbrace{\xi_6 EM_4^{LN}}_{\text{LN Efflux}}$$

$N_{Non,4}^{LN}$ - LnN4Non - Non-cognate naive CD4⁺ T-cell count

$$\frac{d}{dt}(N_{Non,4}^{LN}) = \alpha \left(\underbrace{k_1 N_{Non,4}^B \left(\frac{APC}{APC + hs_1} \right)}_{\text{Cytokine-mediated recruitment}} + \underbrace{\xi_1 N_{Non,4}^B}_{\text{LN Influx}} \right) - \underbrace{\xi_2 N_{Non,4}^{LN}}_{\text{LN Efflux}}$$

$CM_{Non,4}^{LN}$ - LnCM4Non - Non-cognate central memory CD4⁺ T-cell count

$$\frac{d}{dt} (CM_{Non,4}^{LN}) = \alpha \left(\underbrace{k_8 CM_{Non,4}^B \left(\frac{APC}{APC + hs_8} \right)}_{\text{Cytokine-mediated recruitment}} + \underbrace{\xi_4 CM_{Non,4}^B}_{\text{LN Influx}} \right) - \underbrace{\xi_5 CM_{Non,4}^{LN}}_{\text{LN Efflux}}$$

2.3. CD8⁺ T-cells in lymph nodes.

N_8^{LN} - LnN8 - Mtb-specific naive CD8⁺ T-cell count

$$\begin{aligned} \frac{d}{dt} (N_8^{LN}) = & \alpha \left(\underbrace{k_{10} N_8^B \left(\frac{APC}{APC + hs_{10}} \right)}_{\text{Cytokine-mediated recruitment}} + \underbrace{\xi_7 N_8^B}_{\text{LN Influx}} \right) - \underbrace{\xi_8 N_8^{LN}}_{\text{LN Efflux}} \\ & - \underbrace{k_{11} N_8^{LN} APC \left(\frac{E_4^{LN} + w_{P_4} P_4^{LN}}{E_4^{LN} + w_{P_4} P_4^{LN} + hs_{11}} \right)}_{\text{Priming mediated by CD4-related cytokines}} \end{aligned}$$

P_8^{LN} - LnP8 - Mtb-specific precursor CD8⁺ T-cell count

$$\begin{aligned} \frac{d}{dt} (P_8^{LN}) = & \underbrace{k_{11} N_8^{LN} APC \left(\frac{E_4^{LN} + w_{P_4} P_4^{LN}}{E_4^{LN} + w_{P_4} P_4^{LN} + hs_{11}} \right)}_{\text{Priming mediated by CD4-related cytokines}} + \underbrace{k_{12} CM_8^{LN} APC}_{\text{Differentiation from } CM_8^{LN}} \\ & + \underbrace{k_{13} P_8^{LN} \left(1 - \frac{P_8^{LN}}{\rho_1} \right) \left(\frac{APC}{APC + hs_{13}} \right)}_{\text{Cytokine-mediated proliferation}} - \underbrace{k_{15} P_8^{LN} \left(1 - \left\{ \frac{APC}{APC + hs_{14}} \right\} \right)}_{\text{Differentiation to } CM_8^{LN}} \\ & - \underbrace{k_{14} P_8^{LN} \left\{ \frac{APC}{APC + hs_{14}} \right\}}_{\text{Differentiation to } E_8^{LN}} - \underbrace{\mu_7 P_8^{LN}}_{\text{Natural death}} \end{aligned}$$

E_8^{LN} - LnE8 - Mtb-specific effector CD8⁺ T-cell count

$$\frac{d}{dt} (E_8^{LN}) = \underbrace{k_{14} P_8^{LN} \left\{ \frac{APC}{APC + hs_{14}} \right\}}_{\text{Differentiation from } P_8^{LN}} - \underbrace{\xi_9 E_8^{LN}}_{\text{LN Efflux}} - \underbrace{k_{16} E_8^{LN}}_{\text{Differentiation to } EM_8^{LN}}$$

CM_8^{LN} - LnCM8 - Mtb-specific central memory CD8⁺ T-cell count

$$\begin{aligned} \frac{d}{dt} (CM_8^{LN}) = & \alpha \left[\underbrace{k_{17} CM_8^B \left(\frac{APC}{APC + hs_{17}} \right)}_{\text{Cytokine-mediated recruitment}} + \underbrace{\xi_{10} CM_8^B}_{\text{LN Influx}} \right] + \underbrace{k_{15} P_8^{LN} \left(1 - \left\{ \frac{APC}{APC + hs_{14}} \right\} \right)}_{\text{Differentiation from } P_8^{LN}} \\ & - \underbrace{k_{12} CM_8^{LN} APC}_{\text{Cytokine-mediated differentiation to } P_8^{LN}} - \underbrace{\xi_{11} CM_8^{LN}}_{\text{LN Efflux}} \end{aligned}$$

EM_8^{LN} - LnEM8 - Mtb-specific effector memory CD8⁺ T-cell count

$$\frac{d}{dt} (EM_8^{LN}) = \underbrace{k_{16} E_8^{LN}}_{\text{Differentiation from } E_8^{LN}} - \underbrace{\xi_{12} EM_8^{LN}}_{\text{LN Efflux}}$$

$N_{Non,8}^{LN}$ - LnN8Non - Non-cognate Naive CD8⁺ T-cell count

$$\frac{d}{dt} (N_{Non,8}^{LN}) = \alpha \left[\underbrace{k_{10} N_{Non,8}^B \left(\frac{APC}{APC + h s_{10}} \right)}_{\text{Cytokine-mediated recruitment}} + \underbrace{\xi_7 N_{Non,8}^B}_{\text{LN Influx}} \right] - \underbrace{\xi_8 N_{Non,8}^{LN}}_{\text{LN Efflux}}$$

$CM_{Non,8}^{LN}$ - LnCM8Non - Non-cognate central memory CD8⁺ T-cell count

$$\frac{d}{dt} (CM_{Non,8}^{LN}) = \alpha \left[\underbrace{k_{17} CM_{Non,8}^B \left(\frac{APC}{APC + h s_{17}} \right)}_{\text{Cytokine-mediated recruitment}} + \underbrace{\xi_{10} CM_{Non,8}^B}_{\text{LN Influx}} \right] - \underbrace{\xi_{11} CM_{Non,8}^{LN}}_{\text{LN Efflux}}$$

2.4. CD4⁺ T-cells in blood.

N_4^B - Bin4 - Mtb-specific naive CD4⁺ T-cell concentration

$$\frac{d}{dt} (N_4^B) = \underbrace{\lambda S_{N_4}}_{\text{Production in thymus}} + \underbrace{\alpha^{-1} \xi_2 N_4^{LN}}_{\text{LN Efflux}} - \underbrace{k_1 N_4^B \left(\frac{APC}{APC + h s_1} \right)}_{\text{Cytokine-mediated recruitment}} - \underbrace{\xi_1 N_4^B}_{\text{LN Influx}} - \underbrace{\mu_8 N_4^B}_{\text{Natural death}}$$

E_4^B - BIE4 - Mtb-specific effector CD4⁺ T-cell concentration

$$\frac{d}{dt} (E_4^B) = \underbrace{\alpha^{-1} \xi_3 E_4^{LN}}_{\text{LN Efflux}} - \underbrace{\mu_1 E_4^B}_{\text{Natural death}} - \sum_{g \in \text{Granulomas}} \text{Pulled} [E_4^B]_g$$

CM_4^B - BICM4 - Mtb-specific central memory CD4⁺ T-cell concentration

$$\frac{d}{dt} (CM_4^B) = \underbrace{\alpha^{-1} \xi_5 CM_4^{LN}}_{\text{LN Efflux}} - \underbrace{\xi_4 CM_4^B}_{\text{LN Influx}} - \underbrace{k_8 CM_4^B \left(\frac{APC}{APC + h s_8} \right)}_{\text{Cytokine-mediated recruitment}}$$

EM_4^B - BIEM4 - Mtb-specific effector memory CD4⁺ T-cell concentration

$$\frac{d}{dt} (EM_4^B) = \underbrace{\alpha^{-1} \xi_6 EM_4^{LN}}_{\text{LN Efflux}} - \underbrace{\mu_2 EM_4^B}_{\text{Natural death}} - \sum_{g \in \text{Granulomas}} \text{Pulled} [EM_4^B]_g$$

$N_{Non,4}^B$ - Bin4Non - Non-cognate naive CD4⁺ T-cell concentration

$$\frac{d}{dt} (N_{Non,4}^B) = \underbrace{(1 - \lambda) s_{N_4}}_{\text{Production in thymus}} + \underbrace{\alpha^{-1} \xi_2 N_{Non,4}^{LN}}_{\text{LN Efflux}} - \underbrace{k_1 N_{Non,4}^B \left(\frac{APC}{APC + h s_1} \right)}_{\text{Cytokine-mediated recruitment}} - \underbrace{\xi_1 N_{Non,4}^B}_{\text{LN Influx}} - \underbrace{\mu_8 N_{Non,4}^B}_{\text{Natural death}}$$

$E_{Non,4}^B$ - BIE4Non - Non-cognate effector CD4⁺ T-cell concentration

$$\frac{d}{dt} (E_{Non,4}^B) = \underbrace{s_{E_{Non,4}}}_{\text{Production in thymus}} - \underbrace{\mu_4 E_{Non,4}^B}_{\text{Natural death}} - \sum_{g \in \text{Granulomas}} \text{Pulled} [E_{Non,4}^B]_g$$

$CM_{Non,4}^B$ - BICM4Non - Non-cognate central memory CD4⁺ T-cell concentration

$$\frac{d}{dt} (CM_{Non,4}^B) = \underbrace{\alpha^{-1} \xi_5 CM_{Non,4}^{LN}}_{\text{LN Efflux}} - \underbrace{\xi_4 CM_{Non,4}^B}_{\text{LN Influx}} - \underbrace{k_8 CM_{Non,4}^B \left(\frac{APC}{APC + h s_8} \right)}_{\text{Cytokine-mediated recruitment}}$$

$EM_{Non,4}^B$ - BIEM4Non - Non-cognate effector memory CD4⁺ T-cell concentration

$$\frac{d}{dt} (EM_{Non,4}^B) = \underbrace{s_{EM_{Non,4}}}_{\text{Source from thymus}} - \underbrace{\mu_2 EM_{Non,4}^B}_{\text{Natural death}} - \sum_{g \in Granulomas} Pulled [EM_{Non,4}^B]_g$$

2.5. CD8⁺ T-cells in blood.

N_8^B - B1N8 Mtb-specific naive CD8⁺ T-cell concentration

$$\frac{d}{dt} (N_8^B) = \underbrace{\lambda s_{N_8}}_{\text{Source from thymus}} + \underbrace{\alpha^{-1} \xi_8 N_8^{LN}}_{\text{LN efflux}} - \underbrace{k_{10} N_8^B \left(\frac{APC}{APC + h s_{10}} \right)}_{\text{Cytokine-mediated recruitment}} - \underbrace{\xi_7 N_8^B}_{\text{LN Influx}} - \underbrace{\mu_9 N_8^B}_{\text{Natural death}}$$

E_8^B - B1E8 - Mtb-specific effector CD8⁺ T-cell concentration

$$\frac{d}{dt} (E_8^B) = \underbrace{\alpha^{-1} \xi_9 E_8^{LN}}_{\text{LN Efflux}} - \underbrace{\mu_3 E_8^B}_{\text{Natural death}} - \sum_{g \in Granulomas} Pulled [E_8^B]_g$$

CM_8^B - B1CM8 - Mtb-specific central memory CD8⁺ T-cell concentration

$$\frac{d}{dt} (CM_8^B) = \underbrace{\alpha^{-1} \xi_{11} CM_8^{LN}}_{\text{LN Efflux}} - \underbrace{\xi_{10} CM_8^B}_{\text{LN Influx}} - \underbrace{k_{17} CM_8^B \left(\frac{APC}{APC + h s_{17}} \right)}_{\text{Cytokine-mediated recruitment}}$$

EM_8^B - B1EM8 - Mtb-specific Effector memory CD8⁺ T-cell concentration

$$\frac{d}{dt} (EM_8^B) = \underbrace{\alpha^{-1} \xi_{12} EM_8^{LN}}_{\text{LN Efflux}} - \underbrace{\mu_4 EM_8^B}_{\text{Natural death}} - \sum_{g \in Granulomas} Pulled [EM_8^B]_g$$

$N_{Non,8}^B$ - B1N8Non - Non-cognate naive CD8⁺ T-cell concentration

$$\begin{aligned} \frac{d}{dt} (N_{Non,8}^B) = & \underbrace{(1 - \lambda) s_{N_8}}_{\text{Source from thymus}} + \underbrace{\alpha^{-1} \xi_8 N_{Non,8}^{LN}}_{\text{LN Efflux}} \\ & - \underbrace{k_{10} N_{Non,8}^B \left(\frac{APC}{APC + h s_{10}} \right)}_{\text{Cytokine-mediated recruitment}} - \underbrace{\xi_7 N_{Non,8}^B}_{\text{LN Influx}} - \underbrace{\mu_9 N_{Non,8}^B}_{\text{Natural death}} \end{aligned}$$

$E_{Non,8}^B$ - B1E8Non - Non-cognate effector CD8⁺ T-cell concentration

$$\frac{d}{dt} (E_{Non,8}^B) = \underbrace{s_{E_{Non,8}}}_{\text{Source from thymus}} - \underbrace{\mu_3 E_{Non,8}^B}_{\text{Natural death}} - \sum_{g \in Granulomas} Pulled [E_{Non,8}^B]_g$$

$CM_{Non,8}^B$ - B1CM8Non - Non-cognate central memory CD8⁺ T-cell concentration

$$\frac{d}{dt} (CM_{Non,8}^B) = \underbrace{\alpha^{-1} \xi_{11} CM_{Non,8}^{LN}}_{\text{LN Efflux}} - \underbrace{\xi_{10} CM_{Non,8}^B}_{\text{LN Influx}} - \underbrace{k_{17} CM_{Non,8}^B \left(\frac{APC}{APC + h s_{17}} \right)}_{\text{Cytokine-mediated recruitment}}$$

$EM_{Non,8}^B$ - B1EM8Non - Non-cognate effector memory CD8⁺ T-cell concentration

$$\frac{d}{dt} (EM_{Non,8}^B) = \underbrace{s_{EM_{Non,8}}}_{\text{Source from thymus}} - \underbrace{\mu_4 EM_{Non,8}^B}_{\text{Natural death}} - \sum_{g \in Granulomas} Pulled [EM_{Non,8}^B]_g$$

3. COUPLING GRANULOMA EQUATIONS TO LYMPH NODE AND BLOOD EQUATIONS

3.1. Migration of APCs from granulomas to lymph nodes. Each granuloma begins to send a portion of its infected macrophages to the lymph node compartment. We use the sum of infected macrophages M_I and antigen Ag as a proxy for the number of dendritic cells that take Mtb to lung draining lymph nodes. Each granuloma sends APCs to the lymph nodes after time τ_s under the assumption that granulomas do not immediately begin to send dendritic cells to the lymphatic system. We compute APCs sent by granuloma g to the lymph node as:

$$APC \text{ Sent by Granuloma } g \text{ at time } t \text{ post-founding} = \begin{cases} 0 & \text{if } t < \tau_s \\ A_{frac} \cdot (Ag + M_I) & \text{otherwise.} \end{cases}$$

3.2. Granulomas request T-cells, pulling them from the blood compartment. In *HostSim*, a given host begins with 13 granulomas. As infection progresses, each granuloma will begin to attempt to recruit six types of T-cells from the blood compartment: $T_0^4, T_0^8, T_{EM}^4, T_{EM}^8, T_{Non}^4$, and T_{Non}^8 . At each time step, each granuloma will independently request a number of T-cells given below:

- Requests for T_0^4 :

$$Requested [T_0^4] = \underbrace{\alpha_{1a} (w_2 M_I + M_A)}_{\text{Macrophage-mediated request}} + \underbrace{Sr_{1b} \left(\frac{F_\alpha}{F_\alpha + f_8 I_{10} + s_{4b2}} \right)}_{\text{TNF mediated recruitment}}$$

- Requests for T_0^8 :

$$Requested [T_0^8] = \underbrace{\alpha_{1a} (M_A + w_2 M_I)}_{\text{Macrophage-mediated recruitment}} + \underbrace{Sr_{1b} \left(\frac{F_\alpha}{F_\alpha + f_8 I_{10} + s_{4b2}} \right)}_{\text{TNF mediated recruitment}}$$

- Requests for T_{EM}^4 :

$$Requested [T_{EM}^4] = \underbrace{Sr_{4EM} \left(\frac{F_\alpha}{F_\alpha + c_{4EM}} \right)}_{\text{TNF driven recruitment}}$$

- Requests for T_{EM}^8 :

$$Requested [T_{EM}^8] = \underbrace{Sr_{8EM} \left(\frac{F_\alpha}{F_\alpha + h s_{8EM}} \right)}_{\text{TNF-mediated recruitment}}$$

- Requests for T_{Non}^4 :

$$Requested [T_{Non}^4] = \underbrace{Sr_{4Non} \left(\frac{F_\alpha}{F_\alpha + h s_{4Non}} \right)}_{\text{TNF driven recruitment}}$$

- Requests for T_{Non}^8 :

$$Requested [T_{Non}^8] = \underbrace{Sr_{8Non} \left(\frac{F_\alpha}{F_\alpha + h s_{8Non}} \right)}_{\text{TNF driven recruitment}}$$

The blood's response to these requests is to provide lung each granuloma with T-cells. Since we model the T-cell subpopulations in more granularity in the lymph node and blood compartments, we associate the following populations between the terminology of the lymph node and blood equations; and the granuloma equations:

- (1) Granuloma requests for T_0^4 pull from Lymph/blood concentration E_4^B .
- (2) Granuloma requests for T_0^8 pull from Lymph/blood concentration E_8^B .
- (3) Granuloma requests for T_{EM}^4 pull from Lymph/blood concentration EM_4^B .
- (4) Granuloma requests for T_{EM}^8 pull from Lymph/blood concentration EM_8^B .
- (5) Granuloma requests for T_{Non}^4 pull from both Lymph/blood concentrations $E_{Non,4}^B$ and $EM_{Non,4}^B$.
- (6) Granuloma requests for T_{Non}^8 pull from both Lymph/blood concentrations $E_{Non,8}^B$ and $EM_{Non,8}^B$.

For T_0^4, T_0^8, T_{EM}^4 , and T_{EM}^8 , the amount that is pulled from the blood to provide to granuloma g is given by:

$$\underbrace{Pulled [T^*]_g}_{T - \text{Cell count}} = \alpha \cdot \underbrace{Pulled [E^*]_g}_{E - \text{Concentration}} = \begin{cases} Requested [T^*]_g & \text{if } \sum_{\text{Granulomas } j} Requested [T^*]_j < \alpha \cdot E^* \\ Relative Request [T^*]_g & \text{otherwise.} \end{cases}$$

Here, if there are not enough T-cells in the blood to meet the amount requested by the lung granulomas, available T-cells will be divided among the requesting granulomas by using the following relative request. For granuloma g , its relative request is

$$Relative Request [T^*]_g = \frac{\alpha \cdot E^*}{\sum_{\text{All granulomas } j} Request [T^*]_j}.$$

For T_{Non}^4 and T_{Non}^8 , we combine their two associated blood concentrations. For example,

$$\begin{aligned} Pulled [T_{Non}^4]_g &= \alpha \left(Pulled [E_{Non,4}^B]_g + Pulled [EM_{Non,4}^B]_g \right) \\ &= \begin{cases} Requested [T_{Non}^4]_g & \text{if } \sum_{\text{Granulomas } j} Requested [T_{Non}^4]_j < \alpha (E_{Non,4}^B + EM_{Non,4}^B) \\ Relative Request [T_{Non}^4]_g & \text{otherwise} \end{cases} \end{aligned}$$

If the blood has excess T-cells ($E_{Non,4}^B$ and $EM_{Non,4}^B$ in this case), the weight of their contribution to the granulomas is based on their current relative population:

$$\begin{aligned} \alpha \cdot Pulled [E_{Non,4}^B]_g &= Requested [T_{Non}^4]_g \frac{E_{Non,4}^B}{(E_{Non,4}^B + EM_{Non,4}^B)}; \\ \alpha \cdot Pulled [EM_{Non,4}^B]_g &= Requested [T_{Non}^4]_g \frac{EM_{Non,4}^B}{(E_{Non,4}^B + EM_{Non,4}^B)}. \end{aligned}$$

If there are not enough T-cells for the request, the relative requests are calculated similarly and the blood concentration is depleted:

$$\begin{aligned} \alpha \cdot Pulled [E_{Non,4}^B]_g &= \frac{Requested [T_{Non}^4]_g}{\sum_{\text{Granulomas } j} Requested [T_{Non}^4]_j} \frac{E_{Non,4}^B}{(E_{Non,4}^B + EM_{Non,4}^B)}; \\ \alpha \cdot Pulled [EM_{Non,4}^B]_g &= \frac{Requested [T_{Non}^4]_g}{\sum_{\text{Granulomas } j} Requested [T_{Non}^4]_j} \frac{EM_{Non,4}^B}{(E_{Non,4}^B + EM_{Non,4}^B)}. \end{aligned}$$

Pulled concentrations for T_{Non}^8 are calculated in the same way.

4. DISSEMINATION

In *HostSim*, nonsterile granulomas have a chance to disseminate. Each granuloma begins with a local dissemination probability adjustment L_t and nonlocal dissemination probability adjustment N_t when the granuloma is created (either primary at $t = 0$, or at some later time). At each timestep, a dissemination event is determined randomly determining:

$$\{\text{Local dissemination occurs at time } t\} = (L_t + U[0, 1]) > 1$$

and

$$\{\text{Nonlocal dissemination occurs at time } t\} = (N_t + U[0, 1]) > 1$$

At each time step, the probability adjustment of local dissemination increases:

$$L_t = L_{t-1} + \lambda_{local} \left(\frac{[B_I + B_E](t)}{[B_I + B_E](t) + k_{local}} \right).$$

Similarly, nonlocal dissemination probability adjustment builds as

$$N_t = N_{t-1} + \lambda_{nonlocal} \left(\frac{[B_I + B_E](t)}{[B_I + B_E](t) + k_{nonlocal}} \right).$$

A new locally-disseminated granuloma is created with its parameters selected to be normally distributed around the parameter values of the primary granuloma that it disseminated from with $\sigma = 10\%$ of the width of the parameter's range (log-transformed, if the parameter is log-uniformly sampled). This is because of the assumption that nearby lung tissue conditions give rise to similar dynamics. Nonlocally disseminated granulomas are given parameters uniformly sampled out of the ranges presented in Section 5.

Disseminated granulomas begin with one infected macrophage and an average number of intracellular bacteria from its parent (B_I/M_I from the primary granuloma that seeded it).

5. PARAMETER RANGES

In all parameter tables, distributions listed as 'u' are uniformly sampled within the given range, and 'l' are log-uniformly sampled from within the given range. Note that the rate constants k_i in the granuloma compartment and LN/blood compartment are distinct. Certain parameters indicated with * have values computed dynamically by other model components. Also note that for space, we refer to "rates" as shorthand for "rate constants" in all following tables.

5.1. Granuloma parameter ranges.

Parameter	Non-symbolic	Units	Granuloma parameter description	Minimum Value	Maximum Value	Distribution
ψ_C	psiC	-	Antibiotic clearance rate of M_I	0.0001	0.001	u
ψ_S	psiS	-	Survival rate of cleared M_I	0	1	u
C_{eff}	Ceff	-	Effective concentration of antibiotic	*	*	-
α_{4a}	alpha4a	1/day	Macrophage-driven M_R recruitment rate	0.7246	0.9097	u
β	Beta	1/pg	Scaling factor for F_α for M_R activation	8652476.9683	11390938.3080	u
w	w	-	Contribution of B_I to M_R recruitment	0.2601	0.3632	u
w_2	w2	-	Contribution of M_I to M_R recruitment	0.1	0.5	u
w_3	w3	-	Contribution of T_1^4 to M_I apoptosis	0.9572	1.1725	u
Sr_{4b}	Sr4b	1/day	F_α recruitment rate of M_R	400.3165	961.7117	u
f_8	f8	-	Ratio adjustment of I_{10}/F_α on M_R recruitment	0.02568	0.9748	u
f_9	f9	-	Ratio adjustment of I_{10}/F_α for apoptosis rates	0.1	1.0	u
s_{4b}	s4b	pg/mL	Half saturation of F_α on M_R recruitment	43.2587	432.1618	l
k_4	k4	1/day	M_A deactivation by I_{10}	0.06138	0.1816	u
s_8	s8	pg/mL	Half saturation of I_{10} on M_A deactivation	18.2688	156.2867	l
k_2	k2	1/day	M_R infection rate	0.1126	0.4900	u
c_9	c9	count	Half saturation of B_E on M_R infection	1305.8725	8590.4539	u
k_3	k3	1/day	M_R activation rate	0.1080	0.5012	u
f_1	f1	-	Ratio adjustment of I_4/I_γ on M_R activation	132.009929	477.7016	u
s_1	s1	pg/mL	Half saturation of I_γ dependent M_R activation	2.6104	120.4874	l
c_8	c8	count	Half saturation of B_E and B_I on M_R activation	345.3760	681.6706	u

Parameter	Non-symbolic	Units	Granuloma parameter description	Minimum Value	Maximum Value	Distribution
μ_{M_R}	muMR	1/day	M_R death rate	0.004432	0.005695	u
k_{17}	k17	1/day	Maximum rate of M_I bursting	0.03598	0.3017	u
N	N	count	Carrying capacity of Mtb in one M_I	5	40	u
k_{14a}	k14a	1/day	T-cell induced apoptosis rate of M_I	0.2726	0.45	u
c_4	c4	count	Half saturation of T_1^4/M_I on T-cell apoptosis	905.8568	9420.9084	l
k_{14b}	k14b	1/day	F_α -induced apoptosis rate	0.2230	0.4355	u
k_{52}	k52	1/day	Cytotoxic killing rate of M_I	0.08592	0.3519	u
w_1	w1	-	Contribution of T_1^4 to cytotoxic killing	0.2192	0.7413	u
c_{52}	c52	count	Half-saturation of T_C on cytotoxic M_I killing	2588.9284	8088.5625	u
$C_{T_1^4}$	cT1	count	Half saturation of T_1^4 on cytotoxic killing	80.6446	300.4901	u
μ_{M_I}	muMI	1/day	M_I death rate	0.002965	0.003799	u
μ_{M_A}	muMA	1/day	M_A death rate	0.05	0.1	u
α_{1a}	alpha1a	1/day	Macrophage request rate of T_0^4	0.1769	0.4336	u
Sr_{1b}	Sr1b	1/day	F_α dependent T_0^4 recruitment	23767.7947	55329.7803	u
s_{4b2}	s4b2	pg/mL	Half saturation of F_α dependent T_0^4 recruitment	462.3252	1057.4677	l
α_2	alpha2	1/day	Maximum growth rate of T_0^4	0.2444	0.9666	u
c_{15}	c15	count	Half saturation of M_A proliferation of T_0^4	531.8574	9574.4150	u
k_6	k6	1/day	Maximum T_0^4 to T_1^4 differentiation rate	0.01372	0.09727	u
f_7	f7	-	Effect of I_{10} on I_γ induced $T_0^4 \rightarrow T_1^4$ differentiation	5.7185	45.2677	u
k_7	k7	1/day	Maximum rate of $T_0^4 \rightarrow T_2^4$ differentiation	0.2437	0.6525	u
f_2	f2	-	Ratio adjustment of I_γ/I_4 on $T_0^4 \rightarrow T_2^4$ differentiation	0.1938	0.4207	u
s_2	s2	pg/mL	Half saturation of I_4 for $T_0^4 \rightarrow T_2^4$ differentiation	100.3818	964.1090	l
μ_{T_0}	muT0	1/day	CD3 ⁺ primed T-cell death rate	0.1957	0.2474	u
m	m	-	Fraction of differentiating T_0^8 that become T^8 (not T_C)	0.1064	0.9043	u
μ_{T_g}	muTg	1/day	T_C death rate	0.005308	0.01990	l
c	c	pg/mL	Half saturation of I_γ on T_1^4 apoptosis	462.9043	3161.3955	u

Parameter	Non-symbolic	Units	Granuloma parameter description	Minimum Value	Maximum Value	Distribution
$\mu_{T_1^4}$	muT1	1/day	T_1^4 death rate	0.2848	0.3698	u
$\mu_{T_2^4}$	muT2	1/day	T_2^4 death rate	0.2913	0.3678	u
$\mu_{T_C\gamma}$	muTCg	1/day	I_γ driven apoptosis rate of T_C and T^8	0.006021	0.09506	l
c_c	cc	pg/mL	Half saturation of I_γ on T_C and T^8 apoptosis	368.2888	9634.1767	u
μ_{T_C}	muTC	1/day	T_C death rate	0.2570	0.3325	u
α_{30}	alpha30	1/day	Rate of F_α production by M_I	0.04521	0.09694	u
α_{31}	alpha31	1/day	Rate of F_α production by M_A	0.03113	0.09610	u
β_2	beta2	1/pg	Scaling factor of Mtb on F_α production by M_A	10944.8638	13022.3032	u
s_{10}	s10	pg/mL	Half saturation of I_γ on F_α production by M_A	102.8084	1500.4010	u
α_{32}	alpha32	pg/(mL*day)	F_α production rate by T_1^4	0.1843	0.3102	u
α_{33}	alpha33	pg/(mL*day)	F_α production rate by T^8 and T_C	0.1633	0.2997	u
μ_{TNF}	muTNF	1/day	F_α clearance rate	0.9334	1.5101	u
s_g	sg	pg/(mL*day)	I_γ production from other sources (e.g. dendritic cells)	846.3332	9484.1532	l
c_{10}	c10	count	Half saturation of Mtb production from other sources	315332.8289	536623.8464	u
s_7	s7	pg/mL	Half saturation of I_{12} on I_γ production from other sources	891.2493	1151.7672	u
α_{5a}	alpha5a	pg/day	Rate of I_γ production by T_1^4	0.5443	0.8698	u
c_{5a}	c5a	count	Half saturation of M_A on I_γ production by T_1^4	303.7365	687.08808	u
α_{5b}	alpha5b	pg/day	I_γ production by T^8	1.1445	15.1796	u
α_{5c}	alpha5c	pg/day	I_γ production by M_I	0.5592	0.9184	u
c_{5b}	c5b	count	Half saturation of M_A on I_γ production by T^8	235.8238	846.5511	u
α_7	alpha7	pg/day	I_γ production by T_0^4	0.08541	0.3098	u
f_4	f4	-	Adjustment of I_{10}/I_{12} on I_γ production	1.2966	1.6715	u
s_4	s4	pg/mL	Half saturation of I_{12} on I_γ	321.7448	865.2110	u
μ_{I_γ}	muIG	1/day	I_γ clearance rate	6.4475	12.6934	u
α_{23}	alpha23	pg/day	Rate of I_{12} production by M_R	0.003485	0.004652	u
c_{23}	c23	count	Half saturation of Mtb on I_{12} production by M_R	157.08445	525.4198	u

Parameter	Non-symbolic	Units	Granuloma parameter description	Minimum Value	Maximum Value	Distribution
α_8	alpha8	pg/day	Rate of production of I_{12} by M_A	0.3764	0.8612	u
s_{12}	s12	pg/day	Rate of production of I_{12} by dendritic cells	2361.1941	4060.7654	u
c_{230}	c230	count	Half-saturation of Mtb in I_{12} production by DCs	365.6847	761.7284	u
$\mu_{I_{12}}$	mul12	1/day	I_{12} clearance rate	0.9319	1.2420	u
s	s	pg/mL	I_{10} effect on I_{12} production by M_A	191.7181	694.3849	u
δ_7	delta7	pg/day	I_{10} production by M_A	0.1106	0.6169	u
s_6	s6	pg/mL	Half saturation of I_{10} on self	587.3834	858.9373	u
f_6	f6	-	Ratio adjustment of I_γ on I_{10}	0.3011	0.3889	u
α_{16}	alpha16	pg/day	I_{10} production by T_1^4	0.4280	0.6693	u
α_{17}	alpha17	pg/day	I_{10} production by T_2^4	0.4149	0.4786	u
α_{18}	alpha18	pg/day	I_{10} production by T_C and T^8	0.5184	0.6648	u
$\mu_{I_{10}}$	mul10	1/day	I_{10} clearance rate	0.5904	4.007931	u
α_{11}	alpha11	pg/day	I_4 production by T_0^4	0.02944	0.06404	u
α_{12}	alpha12	pg/day	I_4 production by T_2^4	0.02110	0.06407	u
μ_{I_4}	mul4	1/day	I_4 clearance rate	2.3700	3.08806	u
α_{19}	alpha19	1/day	B_I growth rate	0.1081	1.6183	l
α_{20}	alpha20	1/day	B_E growth rate	0.1927	0.6209	u
N_{fracc}	Nfracc	-	Fraction of surviving B_I released by T-cell M_I apop.	0.3074	0.7893	u
N_{fraca}	Nfraca	-	Fraction of surviving B_I released by TNF M_I apop.	0.2680	0.5658	u
k_{15}	k15	1/day	B_E killing rate by M_A	0.04058	0.1173	u
k_{18}	k18	1/day	B_E killing rate by M_R	0.0003264	0.0005505	u
N_{fracd}	Nfracd	-	Fraction of surviving B_I released by M_I natural death.	0.0008721	0.001098	u
μ_{B_I}	muBI	1/day	B_I death rate	0.00002973	0.00004521	u
μ_{B_E}	muBE	1/day	B_E death rate	0.000000002053	0.000000003702	u
Sr_{4Non}	Sr4Non	1/day	Rate of TNF-driven T_{Non}^4 recruitment	158.6388	450.4727	u
hs_{4Non}	hs4Non	pg/mL	Half saturation of TNF in T_{Non}^4 recruitment	6.1706	49.6606	u

Parameter	Non-symbolic	Units	Granuloma parameter description	Minimum Value	Maximum Value	Distribution
μ_{4Non}	mui4Non	1/day	Death rate of T_{Non}^4	0.2631	0.3579	u
Sr_{8Non}	Sr8Non	1/day	Rate of TNF-driven T_{Non}^8 recruitment	156.08771	450.8067	u
hs_{8Non}	hs8Non	pg/day	Half saturation of TNF in T_{Non}^8 recruitment	4.3815	40.4821	u
$\mu_{T_{Non}^8}$	mui8Non	1/day	Death rate of T_{Non}^8	0.2644	0.3543	u
Sr_{4EM}	Sr4EM	1/day	Rate of TNF-driven T_{EM}^4 recruitment	5145.1195	10500.04020	u
hs_{4EM}	hs4EM	pg/mL	Half saturation of TNF in T_{EM}^4 recruitment	100.0	200.0	u
$\mu_{T_{EM}^4}$	mui4EM	1/day	Death rate of T_{EM}^4	0.001368	0.002737	u
k_{31}	k31	1/day	Differentiation rate of $T_{EM}^4 \rightarrow T_1^4$	0.05592	0.1	l
k_{32}	k32	1/day	Differentiation rate of $T_{EM}^4 \rightarrow T_2^4$	0.0006898	0.001086	u
Sr_{EM^8}	Sr8EM	1/day	Rate of TNF-driven T_{EM}^8 recruitment	60.2088	115.9625	u
hs_{EM^8}	hs8EM	pg/mL	Half saturation of TNF in T_{EM}^8 recruitment	100	300	u
$\mu_{T_{EM}^8}$	mui8EM	1/day	Death rate of T_{EM}^8	0.001368	0.002737	u
k_{33}	k33	1/day	Differentiation rate of $T_{EM}^8 \rightarrow T_C$	0.06339	0.1	l
k_{34}	k34	1/day	Differentiation rate of $T_{EM}^8 \rightarrow T^8$	0.05329	0.1	l
τ_s	taus	days	Time that granuloma begins sending APCs to LN	4	17	u
A_{frac}	APCpercent	-	Fraction of M_I and A_g that gets sent as APCs	0.1723	0.2893	u
k_{local}	localDissemCFUHalf	count	Half-saturation of Mtb for local dissemination	10000	50000	u
λ_{local}	localDissemLambda	1/day	Maximum rate of local dissemination	0.0005	0.009	l
$k_{nonlocal}$	nonLocalDissemCFUHalf	count	Half-saturation of Mtb for nonlocal dissemination	5223.04303	10608.01033	u
$\lambda_{nonlocal}$	nonLocalDissemLambda	1/day	Maximum rate of nonlocal dissemination	0.0001	0.002	u
μ_{B_N}	muBN	1/day	Death rate of B_N	0	0	u
C_N	CN	-	Fraction of released B_I that become B_N	0.0003125	0.08172	l
μ_{A_g}	muAg	1/day	Decay rate of Mtb antigen	0.003985	0.02686	u
N_f	Nf	M ϕ /caseum	Macrophage biomass that becomes caseum	13	18	u
N_{Ca}	Nca	1/(day*caseum)	Clearance rate of caseum by neutrophils	0.000001173	0.000008621	u

HOSTSIM EQUATIONS

Initial Condition	Non-Symbolic	Units	Granuloma initial condition description	Min initial value	Max initial value	Distribution
$M_R(0)$	MR	count	Resting macrophages	1.0	5.0	u
$M_I(0)$	MI	count	Infected macrophages	1.0	1.0	u
$M_A(0)$	MA	count	Activated macrophages	1.0	1.0	u
$B_I(0)$	BI	count	Intracellular Mtb	1.0	1.0	u
$B_E(0)$	BE	count	Extracellular Mtb	0	0	u
$B_N(0)$	BN	count	Non-replicating Mtb	0	0	u
$T_0^4(0)$	T40	count	Mtb-specific primed $CD4^+$ T-cells	0	0	u
$T_1^4(0)$	Th1	count	Mtb-specific Th1 $CD4^+$ T-cells	0	0	u
$T_{Non}^4(0)$	CD4Non	count	Nonspecific $CD4^+$ T-cells	0	0	u
$T_2^4(0)$	Th2	count	Mtb-specific Th2 $CD4^+$ T-cells	0	0	u
$T_{EM}^8(0)$	EMCD4	count	Mtb-specific $CD4^+$ effector memory T-cells	0	0	u
$T_0^8(0)$	T80	count	Mtb-specific primed $CD8^+$ T-cells	0	0	u
$T_C(0)$	TC	count	Mtb-specific cytotoxic $CD8^+$ T-cells	0	0	u
$T^8(0)$	T8	count	Mtb-specific effector $CD8^+$ T-cells	0	0	u
$T_{EM}^8(0)$	EMCD8	count	Mtb-specific $CD8^+$ effector memory T-cells	0	0	u
$T_{Non}^8(0)$	CD8Non	count	Nonspecific $CD8^+$ T-cells	0	0	u
$F_\alpha(0)$	TNF	pg/mL	TNF- α concentration	0	0	u
$I_\gamma(0)$	IG	pg/mL	IFN- γ concentration	0	0	u
$I_{12}(0)$	I12	pg/mL	Interleukin 12 concentration	0	0	u
$I_{10}(0)$	I10	pg/mL	Interleukin 10 concentration	0	0	u
$I_4(0)$	I4	pg/mL	Interleukin 4 concentration	0	0	u
$Ca(0)$	CA	mass of $M\phi$	Necrotic tissue mass	0	0	u
$Ag(0)$	AG	mass Ag/Mtb	Antigen mass within granulomas	0	0	u

5.2. **Lymph Node and Blood parameter ranges.** Note that the rate constants k_i in the granuloma compartment and LN/blood compartment are different.

Parameter	non-symbolic	Units	LN/blood parameter description	Lower range	Upper range	Distribution
α	alpha	μL	Conversion from Blood concentration to LN cell counts	360000.0	360000.0	u
Host LNs	hostLn	count	Number of involved LNs in host (Used for initial conditions)	5.0	5.0	-
λ	lambda	-	Frequency of Mtb-specific naive cells in the host	0.0001	0.0001	u
hs_1	hs1	count	Half saturation of APC in N_4^B recruitment to LN	1092.8546	8061.6792	l
hs_{10}	hs10	count	Half saturation of APC in N_8^B recruitment to LN	45.5413	87.7386	u
hs_{11}	hs11	count	Half saturation of $CD4^+$ surrogates in N_8^{LN} priming	12.9592	47.9200	u
hs_{13}	hs13	count	Half saturation of APC in P_8^{LN} proliferation	2684.2696	4055.8241	u
hs_{14}	hs14	count	HS of APC in $P_8^{LN} \rightarrow E_8^{LN}$ and $P_8^{LN} \rightarrow CM_8^{LN}$ differentiation	1904.3871	4144.4838	u
hs_{17}	hs17	count	Half saturation of APC in CM_8^B recruitment to LN	66.1225	403.04107	u
hs_4	hs4	count	Half saturation of APC in P_4^{LN} proliferation	4057.5368	28401.2710	u
hs_5	hs5	count	HS of APC in $P_4^{LN} \rightarrow E_4^{LN}$ and $P_4^{LN} \rightarrow CM_4^{LN}$ differentiation	4134.3783	8006.2583	u
hs_8	hs8	count	Half saturation of APC in $CM_{Non,4}^B$ recruitment to LN	40.4710	57.04244	u
k_1	k1	1/day	Rate of N_4^B recruitment to LN	0.6880	0.8540	u
k_{10}	k10	1/day	Rate of N_8^B recruitment to LN	0.5380	0.6764	u
k_{11}	k11	1/day	Rate of N_8^{LN} priming	0.0001043	0.0002272	u
k_{12}	k12	1/day	CM_8^{LN} reactivation rate	0.0001206	0.0007482	u
k_{13}	k13	1/day	Rate of P_8^{LN} proliferation	0.1961	0.7954	u
k_{14}	k14	1/day	Rate of $P_8^{LN} \rightarrow E_8^{LN}$ differentiation	0.2548	0.7351	u
k_{15}	k15	1/day	Rate of $P_8^{LN} \rightarrow CM_8^{LN}$ differentiation	0.5286	0.8602	u
k_{16}	k16	1/day	Rate of $E_8^{LN} \rightarrow EM_8^{LN}$ differentiation	0.1573	0.8217	u
k_{17}	k17	1/day	Rate of CM_8^B recruitment to LN	0.3483	0.9012	u
k_2	k2	1/day	Rate of $N_4^{LN} \rightarrow P_4^{LN}$ differentiation	0.3279	0.8656	u
k_3	k3	1/day	Rate of $CM_4^{LN} \rightarrow P_4^{LN}$ differentiation	0.02161	0.07979	u
k_4	k4	1/day	Rate of P_4^{LN} proliferation	2.08336	4.7319	u

Parameter	non-symbolic	Units	LN/blood parameter description	Lower range	Upper range	Distribution
k_5	k5	1/day	Rate of $P_4^{LN} \rightarrow E_4^{LN}$ differentiation	0.2721	0.8993	u
k_6	k6	1/day	Rate of $P_4^{LN} \rightarrow CM_4^{LN}$ differentiation	0.3110	0.8645	u
k_7	k7	1/day	Rate of $E_4^{LN} \rightarrow EM_4^{LN}$ differentiation	0.6220	0.9249	u
k_8	k8	1/day	Rate of CM_4^B recruitment to LN	0.02620	0.06984	u
μ_1	mu1	1/day	Death rate of E_4^B	0.2	0.2	u
μ_2	mu2	1/day	Death rate of EM_4^B	0.001368	0.002737	u
μ_3	mu3	1/day	Death rate of E_8^B	0.2	0.2	u
μ_4	mu4	1/day	Death rate of EM_8^B	0.001368	0.002737	u
μ_5	mu5	1/day	Death rate of APC	0.05	0.05	u
μ_6	mu6	1/day	Death rate of P_4^{LN}	0.0005	0.0005	u
μ_7	mu7	1/day	Death rate of P_8^{LN}	0.015	0.015	u
μ_8	mu8	1/day	Death rate of N_4^B	0.3	0.3	u
μ_9	mu9	1/day	Death rate of μ_9^{LN}	0.05	0.05	u
ρ_1	rho1	count	Precursor cell carrying capacity	3467280.5534	14650451.8418	u
w_{P_4}	Wp4	-	Contribution of P_4^{LN} in P_8^{LN} priming	0.7355	0.7355	u
ξ_{11}	xi11	1/day	$CM_8^{LN} \rightarrow CM_8^B$ LN efflux rate	0.2750	1.2233	u
ξ_{12}	xi12	1/day	$EM_8^{LN} \rightarrow EM_8^B$ LN efflux rate	0.2190	1.5214	u
ξ_2	xi2	1/day	$N_4^{LN} \rightarrow N_4^B$ LN efflux rate	1.9942	4.5066	u
ξ_3	xi3	1/day	$E_4^{LN} \rightarrow E_4^B$ LN efflux rate	16.9937	23.6017	u
ξ_5	xi5	1/day	$CM_4^{LN} \rightarrow CM_4^B$ LN Efflux rate	1.07179	3.9185	u
ξ_6	xi6	1/day	$EM_4^{LN} \rightarrow EM_4^B$ LN Efflux rate	0.1567	29.2311	u
ξ_8	xi8	1/day	$N_8^{LN} \rightarrow N_8^B$ LN Efflux rate	0.6344	2.2140	u
ξ_9	xi9	1/day	$E_8^{LN} \rightarrow E_8^B$ LN Efflux rate	1.8734	4.4935	u
ξ_1	xi1	1/day	$N_4^B \rightarrow N_4^{LN}$ LN Influx rate	0.3988	0.9013	u
ξ_4	xi4	1/day	$CM_4^B \rightarrow CM_4^{LN}$ LN Influx rate	0.2143	0.7837	u
ξ_7	xi7	1/day	$N_8^B \rightarrow N_8^{LN}$ LN Influx rate	0.1268	0.4428	u

Parameter	non-symbolic	Units	LN/blood parameter description	Lower range	Upper range	Distribution
ξ_{10}	xi10	1/day	$CM_8^B \rightarrow CM_8^{LN}$ LN Influx rate	0.05500	0.2446	u
s_{N_4}	Sn4	1/day* μL	Production rate of naive $CD4^+$ T-cells	20.1207	47.9125	u
s_{N_8}	Sn8	1/day* μL	Production rate of naive $CD8^+$ T-cells	0.3830	0.4868	u
$s_{E_{Non,4}}$	Senc4	1/day* μL	Production rate of effector non-specific $CD4^+$ T-cells	15.3195	65.7082	u
$s_{EM_{Non,4}}$	Semnc4	1/day* μL	Production rate of effector memory non-specific $CD4^+$ T-cells	6.4041	17.04971	u
$s_{E_{Non,8}}$	Senc8	1/day* μL	Production rate of effector non-specific $CD8^+$ T-cells	10.05032	69.3024	u
$s_{EM_{Non,8}}$	Semnc8	1/day* μL	Production rate of effector memory non-specific $CD8^+$ T-cells	2.8856	5.5002	u

Initial condition	Non-Symbolic	Unit	LN/Blood initial condition description	Lower range	Upper range	Distribution
$APC(0)$	APC	count	Antigen-presenting cells in LN	0	0	u
$N_4^{LN}(0)$	LnN4	count	Mtb-specific naive $CD4^+$ T-cell count in LN	2897.3857	6899.4095	u
$P_4^{LN}(0)$	LnP4	count	Mtb-specific precursor $CD4^+$ T-cell count in LN	0	0	u
$E_4^{LN}(0)$	LnE4	count	Mtb-specific effector $CD4^+$ T-cell count in LN	0	0	u
$CM_4^{LN}(0)$	LnCM4	count	Mtb-specific central memory $CD4^+$ T-cell count in LN	0	0	u
$EM_4^{LN}(0)$	LnEM4	count	Mtb-specific effector memory $CD4^+$ T-cell count in LN	0	0	u
$N_4^B(0)$	BIN4	pg/mL	Mtb-specific naive $CD4^+$ T-cell concentration in blood	*	-	u
$E_4^B(0)$	BIE4	pg/mL	Mtb-specific effector $CD4^+$ T-cell concentration in blood	*	-	u
$CM_4^B(0)$	BICM4	pg/mL	Mtb-specific central memory $CD4^+$ T-cell concentration in blood	*	-	u
$EM_4^B(0)$	BIEM4	pg/mL	Mtb-specific effector memory $CD4^+$ T-cell concentration in blood	*	-	u
$N_8^{LN}(0)$	LnN8	count	Mtb-specific naive $CD8^+$ T-cell count in LN	5515.3085	7010.3557	u
$P_8^{LN}(0)$	LnP8	count	Mtb-specific precursor $CD8^+$ T-cell count in LN	0	0	u
$E_8^{LN}(0)$	LnE8	count	Mtb-specific effector $CD8^+$ T-cell count in LN	0	0	u
$CM_8^{LN}(0)$	LnCM8	count	Mtb-specific central memory $CD8^+$ T-cell count in LN	0	0	u
$EM_8^{LN}(0)$	LnEM8	count	Mtb-specific effector memory $CD8^+$ T-cell count in LN	0	0	u
$N_8^B(0)$	BIN8	pg/mL	Mtb-specific naive $CD8^+$ T-cell concentration in blood	*	-	u
$E_8^B(0)$	BIE8	pg/mL	Mtb-specific effector $CD8^+$ T-cell concentration in blood	*	-	u
$CM_8^B(0)$	BICM8	pg/mL	Mtb-specific central memory $CD8^+$ T-cell concentration in blood	*	-	u
$EM_8^B(0)$	BIEM8	pg/mL	Mtb-specific effector memory $CD8^+$ T-cell concentration in blood	*	-	u
$N_{Non,4}^{LN}(0)$	LnN4Non	count	Nonspecific naive $CD4^+$ T-cell count in LN	28970960.05181	68987195.6821	u
$CM_{Non,4}^{LN}(0)$	LnCM4Non	count	Nonspecific central memory $CD4^+$ T-cell count in LN	13572957.9382	32571421.8047	u
$N_{Non,4}^B(0)$	BIN4Non	pg/mL	Nonspecific naive $CD4^+$ T-cell concentration in blood	402.3744	958.1554	u
$E_{Non,4}^B(0)$	BIE4Non	pg/mL	Nonspecific effector $CD4^+$ T-cell concentration in blood	76.5979	328.5413	u
$CM_{Non,4}^B(0)$	BICM4Non	pg/mL	Nonspecific central memory $CD4^+$ T-cell concentration in blood	188.5133	452.3808	u
$EM_{Non,4}^B(0)$	BIEM4Non	pg/mL	Nonspecific effector memory $CD4^+$ T-cell concentration in blood	160.1049	426.2429	u

Initial condition	Non-Symbolic	Unit	LN/Blood initial condition description	Lower range	Upper range	Distribution
$N_{Non,8}^{LN}(0)$	LnN8Non	count	Nonspecific naive CD8 ⁺ T-cell count in LN	55147570.5476	70096547.5890	u
$CM_{Non,8}^{LN}(0)$	LnCM8Non	count	Nonspecific central memory CD8 ⁺ T-cell count in LN	8169880.9373	30928893.4342	u
$N_{Non,8}^B(0)$	BlN8Non	pg/mL	Nonspecific naive CD8 ⁺ T-cell concentration in blood	765.9384	973.5631	u
$E_{Non,8}^B(0)$	BlE8Non	pg/mL	Nonspecific effector CD8 ⁺ T-cell concentration in blood	50.2516	346.5122	u
$CM_{Non,8}^B(0)$	BlCM8Non	pg/mL	Nonspecific central memory CD8 ⁺ T-cell concentration in blood	113.4705	429.5679	u
$EM_{Non,8}^B(0)$	BlEM8Non	pg/mL	Nonspecific effector memory CD8 ⁺ T-cell concentration in blood	160.3165	305.5719	u

*We assume that at $t = 0$, blood and lymph node concentrations are at equilibrium, so blood concentrations are calculated via their lymph-node starting count via the conversion factor α and HostLNs.

5.3. Pharmacokinetic parameter ranges. Note that all initial conditions are set to 0, because no drug is present in the patient prior to dosing. We uniformly sampled all host-scale PK parameters and host-baseline values of granuloma-scale parameters. Finally, note that we allow for distinct values of V_L , V_{PT} , and V_P within each host. This is because each drug’s PK model is calibrated to a distinct datasets to replicate those experimental trajectories. This is also why we do not include PK parameters in the granuloma and host-scale sensitivity analyses, as this variation is best considered as a form of noise in the model.

Parameter	Non-symbolic	Units	PK parameter description	Scale	Drug	Lower range	Upper range	Distribution
Q_a	QA	$\frac{m}{h}$	Permeability of drug from lung into viable cellular area	Granuloma	INH	1.5	2	u
					RIF	1	1.2	u
					PZA	1	1.2	u
					EMB	0.05	0.09	u
					BDQ	0.5	5	u
					PTM	0.1	0.14	u
					LZD	0.5	0.9	u
					MXF	0.05	0.09	u
P_A	PA	-	Drug partition coefficient between lung and granuloma	Granuloma	INH	0.95	1	u
					RIF	0.4	1.1	u
					PZA	0.4	1.1	u
					EMB	0.8	1.1	u
					BDQ	0.9	10	u
					PTM	2	3	u
					LZD	0.8	1.0	u
					MXF	0.8	1.1	u

Parameter	Non-symbolic	Units	PK parameter description	Scale	Drug	Lower range	Upper range	Distribution
Q_c	QC	$\frac{L}{h}$	Permeability of drug from viable cellular area into caseum	Granuloma	INH	0.08	0.1	u
					RIF	0.8	1	u
					PZA	0.8	1	u
					EMB	0.8	1	u
					BDQ	0.08	0.1	u
					PTM	0.8	1	u
					LZD	0.8	1	u
					MXF	0.08	0.1	u
P_C	PC	-	Drug partition coefficient between viable cellular area and caseum	Granuloma	INH	0.8	0.8	u
					RIF	0.4	1	u
					PZA	0.5	0.8	u
					EMB	0.04	0.8	u
					BDQ	0.012	0.05	u
					PTM	1	1	-
					LZD	0.6	0.8	u
					MXF	0.2	0.55	u
β	beta	-	Drug penetration exponentiation base	Granuloma	INH	0.999	1	u
					RIF	0.0493	0.0533	u
					PZA	0.999	1	u
					EMB	1	1	-
					BDQ	.01	.01	-
					PTM	.0511	.0951	u
					LZD	0.227	0.329	u
					MXF	0.098	0.172	u

Parameter	Non-symbolic	Units	PK parameter description	Scale	Drug	Lower range	Upper range	Distribution
μ_C	muC	$\frac{1}{h \cdot kg}$	Drug degradation and clearance or binding within caseum	Granuloma	INH	0.8	25	u
					RIF	0.08	0.15	u
					PZA	0.08	0.15	u
					EMB	0.01	0.012	u
					BDQ	0.1	0.12	u
					PTM	0.32	0.8	u
					LZD	0.61	3	u
					MXF	0.01	0.05	u
k_d	kd	$\frac{1}{h}$	Transport rate from drug transit compartments	Host	INH	0.5	6	l
					RIF	0.2	0.4	l
					PZA	0.55	0.75	l
					EMB	1	2.75	l
					BDQ	0.03	0.1	l
					PTM	0.35	1.2	l
					LZD	0.48	1.3	l
					MXF	0.025	3.9	l
-	Transit	-	Number of drug transit compartments	Host	INH	2	2	-
					RIF	2	2	-
					PZA	1	1	-
					EMB	1	1	-
					BDQ	1	1	-
					PTM	2	2	-
					LZD	1	1	-
					MXF	1	1	-

Parameter	Non-symbolic	Units	PK parameter description	Scale	Drug	Lower range	Upper range	Distribution
Q	Q	$\frac{L}{h \cdot kg}$	Permeability of drug into peripheral tissue	Host	INH	0.2	7	u
					RIF	1.7	5	u
					PZA	0.1	0.7	u
					EMB	0.04	0.9	u
					BDQ	0.2	0.8	u
					PTM	3	4	u
					LZD	10.36	45	u
					MXF	0.05	40	u
μ_D	muD	$\frac{L}{h \cdot kg}$	Decay rate of drug within lung tissue	Host	INH	0.08	2.5	u
					RIF	0.08	0.15	u
					PZA	0.08	0.15	u
					EMB	0.01	0.012	u
					BDQ	0.01	0.012	u
					PTM	0.32	0.8	u
					LZD	0.61	3	u
					MXF	0.01	0.05	u
μ_{DP}	muDP	$\frac{L}{h \cdot kg}$	Decay rate of drug within peripheral tissue	Host	INH	0.08	2.5	u
					RIF	0.08	0.15	u
					PZA	0.01	0.05	u
					EMB	1	2.5	u
					BDQ	1.5	4	u
					PTM	0.2	0.8	u
					LZD	0.04	0.91	u
					MXF	0.05	0.6	u

Parameter	Non-symbolic	Units	PK parameter description	Scale	Drug	Lower range	Upper range	Distribution
V_P	VP	$\frac{L}{kg}$	Volume of Plasma distribution	Host	INH	0.5	3.0	u
					RIF	0.08	0.6	u
					PZA	0.25	0.75	u
					EMB	2.7	4.5	u
					BDQ	0.05	5	u
					PTM	1	4	u
					LZD	0.16	1.71	u
					MXF	0.65	2.5	u
P_L	PL	-	Drug partition coefficient between lung and plasma	Host	INH	1	1	-
					RIF	0.07	1	u
					PZA	1	1	-
					EMB	12	16	u
					BDQ	10	500	u
					PTM	1.2	20	u
					LZD	1.1	2	u
					MXF	1	9	u
V_{PT}	VPT	$\frac{L}{kg}$	Volume of peripheral tissue distribution	Host	INH	25	40	u
					RIF	0.05	0.2	u
					PZA	0.01	0.05	u
					EMB	0.08	1	u
					BDQ	0.1	50	u
					PTM	3.5	6	u
					LZD	0.2	1.9	u
					MXF	0.03	0.4	u

Parameter	Non-symbolic	Units	PK parameter description	Scale	Drug	Lower range	Upper range	Distribution
V_L	VL	$\frac{L}{kg}$	Volume of lung distribution	Host	INH	0.08	0.08	-
					RIF	0.08	0.6	u
					PZA	0.08	0.6	u
					EMB	1	6	u
					BDQ	1	6	u
					PTM	1	6	u
					LZD	1	6	u
					MXF	0.05	0.6	u